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FAKULTA INFORMATIKY A STATISTIKY
Katedra statistiky a pravděpodobnosti

STATISTIKA

VZORCE PRO 4ST201 a 4ST210

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Popisná statistika

$$p_i = \frac{n_i}{n} \quad \sum_{i=1}^k n_i = n \quad \sum_{i=1}^k p_i = 1 \quad i = 1, 2, \dots, k$$

$$x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$$

$$\tilde{x}_p, 0 < P < 1 \quad \tilde{x}_p = x_{(z_p)}, \quad nP < z_p < nP + 1,$$

$$\tilde{x}_p = \frac{x_{(z_p)} + x_{(z_p+1)}}{2}, \quad nP = z_p$$

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\bar{x} = \frac{\sum_{i=1}^k x_i n_i}{\sum_{i=1}^k n_i}$$

$$\bar{x} = \sum_{i=1}^k x_i p_i$$

$$\bar{x}_H = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}}$$

$$\bar{x}_H = \frac{\sum_{i=1}^k n_i}{\sum_{i=1}^k \frac{n_i}{x_i}}$$

$$\bar{x}_H = \frac{1}{\sum_{i=1}^k \frac{p_i}{x_i}}$$

$$\bar{x}_G = \sqrt[n]{\prod_{i=1}^n x_i} = \sqrt[n]{x_1 \cdot x_2 \cdot \dots \cdot x_n}$$

$$R = x_{\max} - x_{\min}$$

$$s_x^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$s_x^2 = \overline{x^2} - \bar{x}^2 = \frac{\sum_{i=1}^n x_i^2}{n} - \left(\frac{\sum_{i=1}^n x_i}{n} \right)^2$$

$$s_x^2 = \frac{\sum_{i=1}^k (x_i - \bar{x})^2 n_i}{\sum_{i=1}^k n_i}$$

$$s_x^2 = \overline{x^2} - \bar{x}^2 = \frac{\sum_{i=1}^k x_i^2 n_i}{\sum_{i=1}^k n_i} - \left(\frac{\sum_{i=1}^k x_i n_i}{\sum_{i=1}^k n_i} \right)^2$$

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$$s_x^2 = \overline{s^2} + s_{\bar{x}}^2 = \frac{\sum_{i=1}^k s_{ix}^2 n_i}{\sum_{i=1}^k n_i} + \frac{\sum_{i=1}^k (\bar{x}_i - \bar{x})^2 n_i}{\sum_{i=1}^k n_i}$$

$$\bar{x} = \frac{\sum_{i=1}^k \bar{x}_i n_i}{\sum_{i=1}^k n_i}$$

$$s_x^2 = \sum_{i=1}^k s_{ix}^2 p_i + \sum_{i=1}^k (\bar{x}_i - \bar{x})^2 p_i$$

$$\bar{x} = \sum_{i=1}^k \bar{x}_i p_i$$

$$s_x = \sqrt{s_x^2}$$

$$v_x = \frac{s_x}{\bar{x}}$$

Pravděpodobnost

Počet pravděpodobnosti

$$P(A) = \frac{m}{n} \quad P(\bar{A}) = 1 - P(A)$$

$$P(A \cap B) \leq \min(P(A), P(B))$$

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B)$$

Náhodné veličiny

$$P(x) = P(X = x) \quad F(x) = P(X \leq x) = \sum_{x_j \leq x} P(x_j) \quad P(x_1 < X \leq x_2) = \sum_{x_1 < x \leq x_2} P(x) = F(x_2) - F(x_1)$$

$$E(X) = \sum_x x P(x) \quad D(X) = E(X^2) - [E(X)]^2 = \sum_x x^2 P(x) - \left[\sum_x x P(x) \right]^2$$

$$f(x) = F'(x) \quad \int_{-\infty}^{\infty} f(x) dx = 1 \quad F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt$$

$$P(x_1 < X \leq x_2) = \int_{x_1}^{x_2} f(x) dx = F(x_2) - F(x_1)$$

$$x_p, 0 < p < 1 \quad F(x_p) = p \quad x_p = F^{-1}(p)$$

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx \quad D(X) = \int_{-\infty}^{\infty} x^2 f(x) dx - \left[\int_{-\infty}^{\infty} x f(x) dx \right]^2 \quad \sigma = \sqrt{D(X)}$$

Pravděpodobnostní rozdělení

Alternativní rozdělení $A(\pi)$

$$P(x) = \pi^x (1-\pi)^{1-x} \quad x = 0, 1, \quad 0 < \pi < 1$$

$$E(X) = \pi \quad D(X) = \pi(1-\pi)$$

Binomické rozdělení $Bi(n, \pi)$

$$P(x) = \binom{n}{x} \pi^x (1-\pi)^{n-x} \quad x = 0, 1, 2, \dots, n, \quad n \in \mathbb{N}, 0 < \pi < 1$$

$$E(X) = n\pi \quad D(X) = n\pi(1-\pi)$$

Poissonovo rozdělení $Po(\lambda)$

$$P(x) = \frac{\lambda^x}{x!} e^{-\lambda} \quad x = 0, 1, \dots, \lambda > 0,$$

$$E(X) = \lambda \quad D(X) = \lambda$$

Hypergeometrické rozdělení $Hg(M, N, n)$

$$P(x) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}, x = \max(0, M-N+n), \dots, \min(M, n), 1 \leq n < N, 1 \leq M < N$$

$$E(X) = n \frac{M}{N} \quad D(X) = n \frac{M}{N} \left(1 - \frac{M}{N}\right) \frac{N-n}{N-1}$$

Normované normální rozdělení $N(0,1)$

$$U = \frac{X - \mu}{\sigma} \quad -\infty < u < \infty \quad E(U) = 0 \quad D(U) = 1$$

$$\Phi(u) = 1 - \Phi(-u) \quad u_p = -u_{1-p}$$

Normální rozdělení $N(\mu, \sigma^2)$

$$-\infty < x < \infty, -\infty < \mu < \infty, \sigma^2 > 0 \quad E(X) = \mu \quad D(X) = \sigma^2$$

$$u = \frac{x - \mu}{\sigma} \quad F(x) = \Phi(u) = \Phi\left(\frac{x - \mu}{\sigma}\right) \quad x_p = \mu + \sigma u_p$$

$$P(x_1 < X \leq x_2) = P\left(\frac{x_1 - \mu}{\sigma} < \frac{X - \mu}{\sigma} \leq \frac{x_2 - \mu}{\sigma}\right) = P(u_1 < U \leq u_2) = \Phi(u_2) - \Phi(u_1)$$

Chi-kvadrát rozdělení $\chi^2(\nu)$ $x > 0, \nu \in N$

Rozdělení t (Studentovo) $t(\nu)$ $-\infty < x < \infty, \nu \in N$ $t_p(\nu) = -t_{1-p}(\nu)$

F rozdělení (Fisherovo – Snedecorovo) $F(\nu_1, \nu_2)$ $x > 0, \nu_1, \nu_2 \in N$

$$F_p(\nu_1, \nu_2) = \frac{1}{F_{1-p}(\nu_2, \nu_1)}$$

Matematická statistika

$$S = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}}$$

Bodové a intervalové odhady parametrů (teoretické intervaly spolehlivosti)

střední hodnota $\hat{\mu} = \bar{X}$ $\hat{N}\mu = N\bar{X}$

normální rozdělení

a) σ^2 známý

$$P\left(\bar{X} - u_{1-\alpha/2} \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + u_{1-\alpha/2} \frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha$$

$$P\left(\bar{X} - u_{1-\alpha} \frac{\sigma}{\sqrt{n}} < \mu\right) = 1 - \alpha, \quad P\left(\mu < \bar{X} + u_{1-\alpha} \frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha$$

b) σ^2 neznámý

$$P\left(\bar{X} - t_{1-\alpha/2} \frac{S}{\sqrt{n}} < \mu < \bar{X} + t_{1-\alpha/2} \frac{S}{\sqrt{n}}\right) = 1 - \alpha \quad T \sim t(n-1)$$

$$P\left(\bar{X} - t_{1-\alpha} \frac{S}{\sqrt{n}} < \mu\right) = 1 - \alpha, \quad P\left(\mu < \bar{X} + t_{1-\alpha} \frac{S}{\sqrt{n}}\right) = 1 - \alpha$$

obecné rozdělení, σ^2 neznámý, velký výběr ($n > 30$)

$$P\left(\bar{X} - u_{1-\alpha/2} \frac{S}{\sqrt{n}} < E(X) < \bar{X} + u_{1-\alpha/2} \frac{S}{\sqrt{n}}\right) = 1 - \alpha$$

$$P\left(\bar{X} - u_{1-\alpha} \frac{S}{\sqrt{n}} < E(X)\right) = 1 - \alpha, \quad P\left(E(X) < \bar{X} + u_{1-\alpha} \frac{S}{\sqrt{n}}\right) = 1 - \alpha$$

rozptyl σ^2 (normální rozdělení) $\hat{\sigma}^2 = S^2$

Parametr π alternativního rozdělení (odhad relativní četnosti základního souboru)

$\hat{\pi} = P$ $\hat{N}\pi = NP$

$$P\left(P - u_{1-\alpha/2} \sqrt{\frac{P(1-P)}{n}} < \pi < P + u_{1-\alpha/2} \sqrt{\frac{P(1-P)}{n}}\right) = 1 - \alpha$$

$$P\left(P - u_{1-\alpha} \sqrt{\frac{P(1-P)}{n}} < \pi\right) = 1 - \alpha \quad P\left(\pi < P + u_{1-\alpha} \sqrt{\frac{P(1-P)}{n}}\right) = 1 - \alpha$$

Testování statistických hypotéz

Střední hodnota normálního rozdělení

H ₀	H ₁	Testové kritérium	Kritický obor
$\mu = \mu_0$	$\mu > \mu_0$ $\mu < \mu_0$ $\mu \neq \mu_0$	σ^2 známý $U = \frac{\bar{X} - \mu_0}{\sigma} \sqrt{n} \quad U \sim N(0,1)$	$W_\alpha = \{u; u \geq u_{1-\alpha}\}$ $W_\alpha = \{u; u \leq -u_{1-\alpha}\}$ $W_\alpha = \{u; u \geq u_{1-\alpha/2}\}$
		σ^2 neznámý $T = \frac{\bar{X} - \mu_0}{S} \sqrt{n} \quad T \sim t(n-1)$	$W_\alpha = \{t; t \geq t_{1-\alpha}\}$ $W_\alpha = \{t; t \leq -t_{1-\alpha}\}$ $W_\alpha = \{t; t \geq t_{1-\alpha/2}\}$

Střední hodnota, obecné rozdělení, velký výběr

H ₀	H ₁	Testové kritérium	Kritický obor
$E(X) = \mu_0$	$E(X) > \mu_0$ $E(X) < \mu_0$ $E(X) \neq \mu_0$	σ^2 neznámý ($n > 30$) $U = \frac{\bar{X} - \mu_0}{S} \sqrt{n} \quad U \approx N(0,1)$	$W_\alpha = \{u; u \geq u_{1-\alpha}\}$ $W_\alpha = \{u; u \leq -u_{1-\alpha}\}$ $W_\alpha = \{u; u \geq u_{1-\alpha/2}\}$

Parametr π alternativního rozdělení (velké výběry)

H ₀	H ₁	Testové kritérium	Kritický obor
$\pi = \pi_0$	$\pi > \pi_0$ $\pi < \pi_0$ $\pi \neq \pi_0$	$U = \frac{P - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \quad U \approx N(0,1)$	$W_\alpha = \{u; u \geq u_{1-\alpha}\}$ $W_\alpha = \{u; u \leq -u_{1-\alpha}\}$ $W_\alpha = \{u; u \geq u_{1-\alpha/2}\}$

Rovnost středních hodnot dvou rozdělení

velké nezávislé výběry

H ₀	H ₁	Testové kritérium	Kritický obor
$E(X_1) = E(X_2)$ $E(X_1) - E(X_2) = 0$	$E(X_1) > E(X_2)$ $E(X_1) < E(X_2)$ $E(X_1) \neq E(X_2)$	σ_1^2 a σ_2^2 neznámé $U = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \quad U \approx N(0,1)$	$W_\alpha = \{u; u \geq u_{1-\alpha}\}$ $W_\alpha = \{u; u \leq -u_{1-\alpha}\}$ $W_\alpha = \{u; u \geq u_{1-\alpha/2}\}$

závislé výběry z normálního rozdělení (párový t -test)

H ₀	H ₁	Testové kritérium	Kritický obor
$\mu_1 = \mu_2$ $\mu_1 - \mu_2 = 0$	$\mu_1 > \mu_2$ $\mu_1 < \mu_2$ $\mu_1 \neq \mu_2$	$T = \frac{\sqrt{n}\bar{D}}{S_D} \quad T \sim t(n-1)$ $D_i = X_{1i} - X_{2i}, i = 1, 2, \dots, n$	$W_\alpha = \{t; t \geq t_{1-\alpha}\}$ $W_\alpha = \{t; t \leq -t_{1-\alpha}\}$ $W_\alpha = \{t; t \geq t_{1-\alpha/2}\}$

Chí-kvadrát test dobré shody

H ₀ a H ₁	Testové kritérium	Kritický obor
H ₀ : $\pi_j = \pi_{0,j} \quad j = 1, \dots, k$ H ₁ : non H ₀	$G = \sum_{j=1}^k \frac{(n_j - n\pi_{0,j})^2}{n\pi_{0,j}} \quad G \approx \chi^2(k-1)$	$W_\alpha = \{g; g \geq \chi^2_{1-\alpha}\}$ $n\pi_j \geq 5$

Analyza závislosti

Kontingenční tabulka (r x s)

$$\sum_{i=1}^r \sum_{j=1}^s n_{ij} = \sum_{i=1}^r n_{i+} = \sum_{j=1}^s n_{+j} = n \quad n'_{ij} = \frac{n_{i+} n_{+j}}{n} \quad n'_{ij} \geq 5$$

H ₀	H ₁	Testové kritérium	Kritický obor
znaky jsou nezávislé	non H ₀	$G = \sum_{i=1}^r \sum_{j=1}^s \frac{(n_{ij} - n_{i+} n_{+j} / n)^2}{n_{i+} n_{+j} / n} \quad G \approx \chi^2((r-1)(s-1))$	$W_\alpha = \{g; g \geq \chi^2_{1-\alpha}\}$

$$C = \sqrt{\frac{G}{G+n}}$$

$$V = \sqrt{\frac{G}{n(m-1)}}, \quad m = \min(r, s)$$

Tabulka 2 · 2

$$G = \frac{n(n_{11}n_{22} - n_{12}n_{21})^2}{n_{1+}n_{+1}n_{2+}n_{+2}}$$

Analýza rozptylu

$$S_y = \sum_{j=1}^k \sum_{i=1}^{n_j} (y_{ji} - \bar{y})^2 = S_{y.m} + S_{y.v} \quad S_{y.m} = \sum_{j=1}^k (\bar{y}_j - \bar{y})^2 n_j \quad S_{y.v} = \sum_{j=1}^k \sum_{i=1}^{n_j} (y_{ji} - \bar{y}_j)^2$$

$$P^2 = \frac{S_{y.m}}{S_y}$$

H ₀	H ₁	Testové kritérium	Kritický obor
$\mu_1 = \mu_2 = \dots = \mu_k$	non H ₀	$F = \frac{\frac{S_{y.m}}{k-1}}{\frac{S_{y.v}}{n-k}} \quad F \sim F(k-1, n-k)$	$W_\alpha = \{F; F \geq F_{1-\alpha}\}$

Regrese a korelace

regresní přímka $y = \beta_0 + \beta_1 x + \varepsilon$,

$$Y = b_0 + b_1 x, \quad \hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x \quad \text{minimum}_{\beta_0, \beta_1} \sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_i)^2$$

$$s_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n} = \bar{xy} - \bar{x} \cdot \bar{y}$$

$$b_1 = \hat{\beta}_1 = \frac{n \sum y_i x_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2} = \frac{\bar{xy} - \bar{x} \cdot \bar{y}}{x^2 - \bar{x}^2} = \frac{s_{xy}}{s_x^2}$$

$$b_0 = \hat{\beta}_0 = \frac{\sum y_i \sum x_i^2 - \sum y_i x_i \sum x_i}{n \sum x_i^2 - (\sum x_i)^2} = \bar{y} - \hat{\beta}_1 \bar{x}$$

Jiné regresní funkce $Y = b_0 + b_1 x + b_2 x^2, \quad \hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x + \hat{\beta}_2 x^2$

$$Y = b_0 + b_1 x_1 + b_2 x_2 + \dots + b_k x_k, \quad \hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \dots + \hat{\beta}_k x_k$$

$$S_y = \sum_{i=1}^n (y_i - \bar{y})^2$$

$$S_T = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2$$

$$S_R = \sum_{i=1}^n (y_i - \hat{y}_i)^2 = \sum_{i=1}^n \hat{\varepsilon}_i^2$$

$$S_y = S_T + S_R$$

$$s_R^2 = \frac{S_R}{n-p}$$

$$s_R = \sqrt{\frac{S_R}{n-p}} = \sqrt{s_R^2}$$

$$R^2 = I^2 = \frac{S_T}{S_y}$$

$$I_{ADJ}^2 = R_{ADJ}^2 = 1 - (1 - I^2) \frac{n-1}{n-p}$$

Test hypotézy o regresním parametru

H ₀	H ₁	Testové kritérium	Kritický obor
$\beta_j = 0$	$\beta_j \neq 0$	$T = \frac{\hat{\beta}_j}{s_{\hat{\beta}_j}} \quad T \sim t(n-p)$	$W_\alpha = \{t; t \geq t_{1-\alpha/2}\}$

Test o modelu $p = k + 1$

H ₀	H ₁	Testové kritérium	Kritický obor
$\beta_0 = c$ $\beta_1 = 0$... $\beta_k = 0$	non H ₀	$F = \frac{\frac{S_T}{p-1}}{\frac{S_R}{n-p}} \quad F \sim F(p-1, n-p)$	$W_\alpha = \{F; F \geq F_{1-\alpha}\}$

korelační koeficient

$$r_{yx} = r_{xy} = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i}{\sqrt{n \sum_{i=1}^n x_i^2 - (\sum_{i=1}^n x_i)^2} \sqrt{n \sum_{i=1}^n y_i^2 - (\sum_{i=1}^n y_i)^2}} = \frac{\overline{xy} - \bar{x} \bar{y}}{\sqrt{(\overline{x^2} - \bar{x}^2)(\overline{y^2} - \bar{y}^2)}} = \frac{s_{xy}}{s_x s_y}$$

H ₀	H ₁	Testové kritérium	Kritický obor
$\rho_{XY} = 0$	$\rho_{XY} \neq 0$	$T = \frac{r_{xy} \sqrt{n-2}}{\sqrt{1-r_{xy}^2}} \quad T \sim t(n-2)$	$W_\alpha = \{t; t \geq t_{1-\alpha/2}\}$

Časové řady

$$\bar{y} = \frac{\sum_{t=1}^T y_t}{T} \quad \bar{y} = \frac{\frac{y_1 + y_2}{2} + \frac{y_2 + y_3}{2} + \dots + \frac{y_{T-1} + y_T}{2}}{T-1} = \frac{\frac{1}{2}y_1 + \sum_{t=2}^{T-1} y_t + \frac{1}{2}y_T}{T-1}$$

$$\bar{y} = \frac{\frac{y_1 + y_2}{2}d_1 + \frac{y_2 + y_3}{2}d_2 + \dots + \frac{y_{T-1} + y_T}{2}d_{T-1}}{d_1 + d_2 + \dots + d_{T-1}}$$

$$\Delta y_t = y_t - y_{t-1} \quad \bar{\Delta y} = \frac{\sum_{t=2}^T \Delta y_t}{T-1} = \frac{y_T - y_1}{T-1}$$

$$k_t = \frac{y_t}{y_{t-1}} \quad \bar{k} = \sqrt[T]{k_2 \cdot k_3 \cdot \dots \cdot k_T} = \sqrt[T-1]{\frac{y_T}{y_1}} \quad \delta_t = k_t - 1 \quad \bar{\delta} = \bar{k} - 1$$

Dekompozice časové řady

$$y_t = T_t + C_t + S_t + \varepsilon_t \quad y_t = T_t \cdot C_t \cdot S_t \cdot \varepsilon_t$$

Modelování trendu

Trendové funkce

$$T_t = \beta_0 + \beta_1 t \quad \hat{T}_t = \hat{\beta}_0 + \hat{\beta}_1 t \quad T_t = \beta_0 + \beta_1 t + \beta_2 t^2 \quad \hat{T}_t = \hat{\beta}_0 + \hat{\beta}_1 t + \hat{\beta}_2 t^2$$

$$MSE = \frac{\sum_{t=1}^T (y_t - \hat{y}_t)^2}{T} \quad DW = \frac{\sum_{t=2}^T (\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1})^2}{\sum_{t=1}^T \hat{\varepsilon}_t^2}$$

Klouzavé průměry

$$m = 2p + 1 \quad \hat{T}_t = \bar{y}_t = \frac{y_{t-p} + \dots + y_{t-1} + y_t + y_{t+1} + \dots + y_{t+p}}{m}$$

$$m = 2p \quad \hat{T}_t = \bar{y}_t = \frac{1}{2m} (y_{t-p} + 2y_{t-p+1} + \dots + 2y_{t-1} + 2y_t + 2y_{t+1} + \dots + 2y_{t+p-1} + y_{t+p})$$

Modelování sezónnosti

Regresní metoda s umělými proměnnými (lineární trend, sezónnost délky 4)

$$y_t = T_t + S_t + \varepsilon_t = \beta_0 + \beta_1 t + \gamma_1 D_{1t} + \gamma_2 D_{2t} + \gamma_3 D_{3t} + \gamma_4 D_{4t} + \varepsilon_t$$

$$y_t^* = \beta_0^* + \beta_1^* t + \gamma_1^* D_{1t} + \gamma_2^* D_{2t} + \gamma_3^* D_{3t} + \varepsilon_t$$

$$\hat{s} = \frac{\hat{\gamma}_1^* + \hat{\gamma}_2^* + \hat{\gamma}_3^*}{4} \quad \hat{S}_j = \hat{\gamma}_j = \hat{\gamma}_j^* - \hat{s}, \quad j = 1, 2, 3 \quad \hat{S}_4 = \hat{\gamma}_4 = -\hat{s} \quad \hat{\beta}_0 = \hat{\beta}_0^* + \hat{s}$$

$$\sum_{j=1}^4 \hat{S}_j = 0$$

Indexní analýza

Individuální indexy jednoduché

$$p = \frac{Q}{q} \quad IQ = Iq \cdot Ip$$

$$Ip = \frac{p_1}{p_0} \quad \Delta p = p_1 - p_0 \quad Iq = \frac{q_1}{q_0} \quad \Delta q = q_1 - q_0 \quad IQ = \frac{Q_1}{Q_0} \quad \Delta Q = Q_1 - Q_0$$

$$I_{t/1} = \frac{y_t}{y_1} = I_{2/1} I_{3/2} \dots I_{t/t-1} \quad I_{t/t-1} = \frac{y_t}{y_{t-1}} = \frac{I_{t/1}}{I_{t-1/1}}$$

Individuální indexy složené

$$I_{\Sigma q} = \frac{\sum q_1}{\sum q_0} = \frac{\sum Iq \cdot q_0}{\sum q_0} = \frac{\sum q_1}{\sum \frac{q_1}{Iq}} \quad \Delta_{\Sigma q} = \sum q_1 - \sum q_0$$

$$I_{\Sigma Q} = \frac{\sum Q_1}{\sum Q_0} = \frac{\sum p_1 q_1}{\sum p_0 q_0} = \frac{\sum IQ \cdot Q_0}{\sum Q_0} = \frac{\sum Q_1}{\sum \frac{Q_1}{IQ}} \quad \Delta_{\Sigma Q} = \sum Q_1 - \sum Q_0$$

$$\bar{I}p = \frac{\bar{p}_1}{\bar{p}_0} = \frac{\frac{\sum Q_1}{\sum q_1}}{\frac{\sum Q_0}{\sum q_0}} = \frac{\frac{\sum p_1 q_1}{\sum q_1}}{\frac{\sum p_0 q_0}{\sum q_0}} = \frac{\frac{\sum Q_1}{p_1}}{\frac{\sum Q_0}{p_0}} \quad \Delta \bar{p} = \bar{p}_1 - \bar{p}_0 = \frac{\sum p_1 q_1}{\sum q_1} - \frac{\sum p_0 q_0}{\sum q_0}$$

Souhrnné indexy

$$Ip^{(L)} = \frac{\sum p_1 q_0}{\sum p_0 q_0} = \frac{\sum Ip \cdot p_0 q_0}{\sum p_0 q_0} = \frac{\sum Ip \cdot Q_0}{\sum Q_0} \quad Ip^{(P)} = \frac{\sum p_1 q_1}{\sum p_0 q_1} = \frac{\sum p_1 q_1}{\sum \frac{p_1 q_1}{Ip}} = \frac{\sum Q_1}{\sum \frac{Q_1}{Ip}}$$

$$Ip^{(F)} = \sqrt{Ip^{(L)} \cdot Ip^{(P)}}$$

$$Iq^{(L)} = \frac{\sum p_0 q_1}{\sum p_0 q_0} = \frac{\sum Iq \cdot p_0 q_0}{\sum p_0 q_0} = \frac{\sum Iq \cdot Q_0}{\sum Q_0} \quad Iq^{(P)} = \frac{\sum p_1 q_1}{\sum p_1 q_0} = \frac{\sum p_1 q_1}{\sum \frac{p_1 q_1}{Iq}} = \frac{\sum Q_1}{\sum \frac{Q_1}{Iq}}$$

$$Iq^{(F)} = \sqrt{Iq^{(L)} \cdot Iq^{(P)}}$$